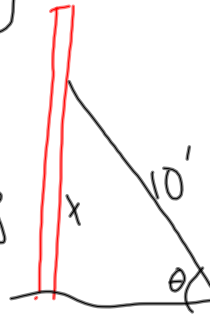


3.4/31)

f/degree
rate of
chg of
x wrt
 θ
=



$$\sin \theta = \frac{x}{10}$$

$$x = 10 \sin \theta$$

$$x = f(\theta)$$

$$x = 10 \sin \theta$$

$$\text{inst r.o.c} = f'(\theta) = \frac{dx}{d\theta}$$

$$x = 10 \sin \theta$$

$$\frac{dx}{d\theta} = 10 \frac{d}{d\theta}(\sin \theta) = 10 \cos \theta$$

FCD (fine calculus detail...)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (\text{when } x \text{ is in radians})$$

or
.0174

Therefore

$$\frac{d}{dx}(\sin x) = \cos x$$

or

$$\frac{dx}{d\theta} = 10 \cos \theta$$

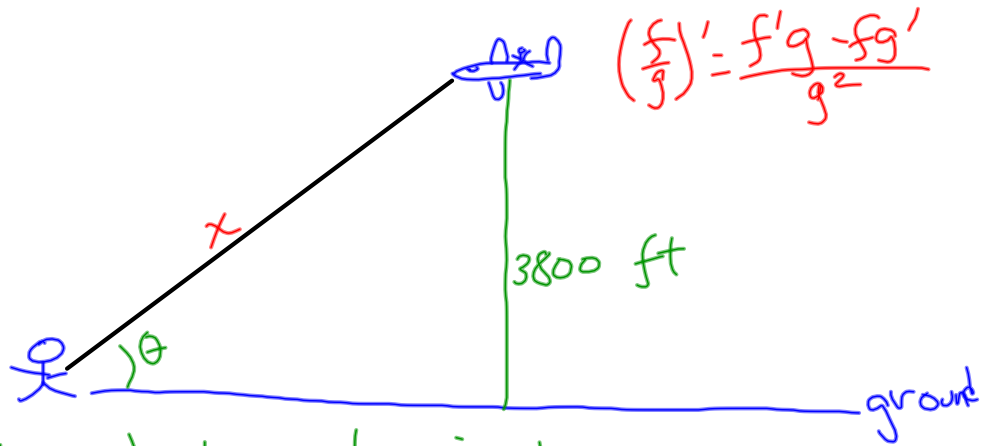
we want
 $\frac{dx}{d\theta}$ when $\theta = 60^\circ$

$$\begin{aligned} \frac{dx}{d\theta} &= 10 \cos(60^\circ) \quad \theta = 60^\circ = \frac{\pi}{3} \text{ radians} \\ &= 10 \cos\left(\frac{\pi}{3}\right) = 10\left(\frac{1}{2}\right) = 5 \text{ ft/radian} \end{aligned}$$

$$5 \frac{\text{ft}}{\text{radian}} \cdot \left(\frac{\pi}{180}\right) \frac{\text{radians}}{\text{degree}}$$

$$= \frac{5\pi}{180} \frac{\text{ft}}{\text{degree}} = .087 \frac{\text{ft}}{\text{deg}}$$

32



At what rate is distance x changing w.r. to θ when $\theta = 30^\circ$
express answer in ft/degree.

$$\sin \theta = \frac{3800}{x}; \text{ so } x = \frac{3800}{\sin \theta}$$

$$\therefore \frac{dx}{d\theta} = \frac{\left(\frac{d}{d\theta}(3800)\right)(\sin \theta) - (3800)\left(\frac{d}{d\theta} \sin \theta\right)}{(\sin \theta)^2}$$

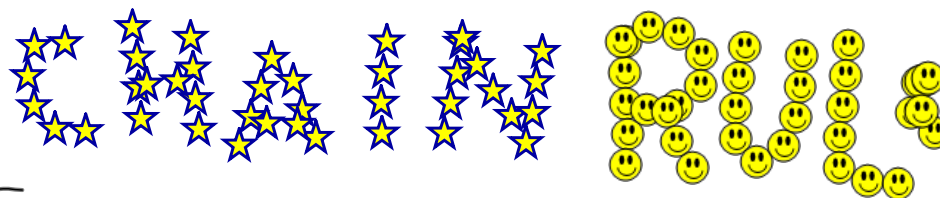
$$\frac{dx}{d\theta} = \frac{0 \cdot \sin \theta - 3800 \cos \theta}{\sin^2 \theta}$$

$$\text{at } \theta = 30^\circ \Rightarrow \frac{dx}{d\theta} = \frac{-3800 \left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{4}} \\ = -7600\sqrt{3} \text{ ft/radian}$$

$$\text{or } \frac{-7600\sqrt{3} \pi}{180} \text{ ft/degree}$$

$$\frac{-7600\sqrt{3} \pi}{180}$$

3.5



Chain Rule: used for
Composition of functions

$$\frac{d}{dx} \left(\frac{3800}{\sin x} \right) = \frac{d}{dx} \left(3800 (\sin x)^{-1} \right)$$

$$x \mapsto \sin x \mapsto \frac{1}{\sin x}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Examples: $f(x) = x^2$

$$g(x) = x^3$$

$$f'(g(x)) \cdot g'(x)$$

mult
out

$$f(g(x)) \stackrel{\text{chain rule}}{=} 2(x^3) \cdot (3x^2)$$

$$= 6x^5$$

$$(x^3)^2$$

$$\Downarrow$$
$$x^6$$

$$\frac{d}{dx}(x^6) = 6x^5$$

$$\frac{d}{dx} \left((x^2+1)^{124} \right)$$

$$f(x) = x^{124}$$

$$g(x) = x^2 + 1$$

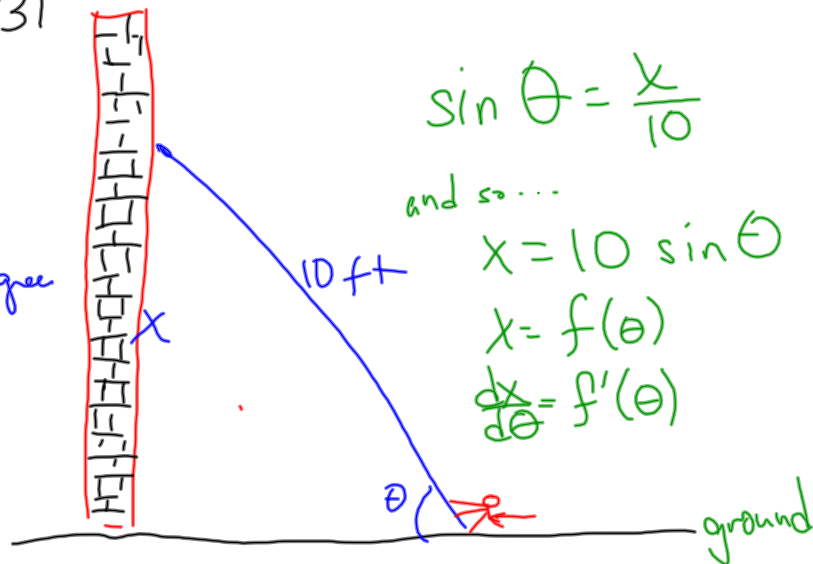
$$x \mapsto x^2 + 1 \mapsto \boxed{}^{124}$$

$$\frac{d}{dx} = f'(g(x)) \cdot g'(x)$$

$$124(x^2+1)^{123}(2x)$$

3.4/31

find
 $\frac{dx}{d\theta}$ in
ft/degree



$$\sin \theta = \frac{x}{10}$$

and so...

$$x = 10 \sin \theta$$

$$x = f(\theta)$$

$$\frac{dx}{d\theta} = f'(\theta)$$

$$x = 10 \sin \theta$$

$$\frac{dx}{d\theta} = 10 \frac{d}{d\theta} \sin \theta = 10 \cos \theta$$

$$\frac{dx}{d\theta} = 10 \cos \theta$$

$$\left. \frac{dx}{d\theta} \right|_{\theta=60^\circ} = 10 \cos 60^\circ = 5$$

interlude

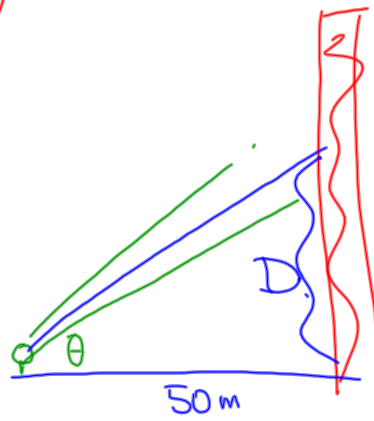
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

(when x is
in radians
0.0174

$$\frac{2\pi \text{ radian}}{360 \text{ degree}} = 0.0174$$

So $\frac{d}{dx}(\sin x) = \cos x$ ONLY WHEN x is in radians

2.4
33)



find the rate
at which D
chgs with respect
to θ
when $\theta = 45^\circ$

express ans in
m/deg

$$\tan \theta = \frac{D}{50}$$

$$D = 50 \tan \theta$$

$$\frac{dD}{d\theta} = 50 \sec^2 \theta$$

$$\text{at } \theta = 45^\circ$$

$$\frac{dD}{d\theta} = 50 \left(\frac{\sqrt{2}}{2} \right)^2 = 100 \text{ m/rad}$$

$$\frac{100 \text{ m}}{\text{rad}} \cdot \frac{\pi \text{ rad}}{180 \text{ deg}} = \frac{100\pi}{180} \text{ m/deg}$$

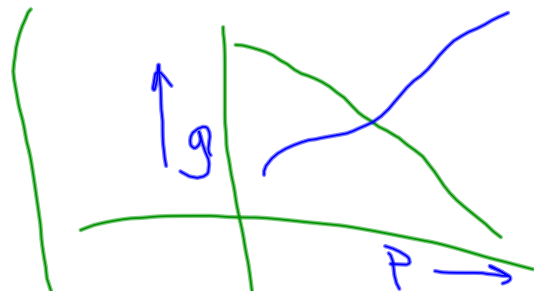
3. ~~3.1~~ 3.6) ~~and~~
given that

$$g = f(p)$$

gallons price

$$\left. \frac{dg}{dp} \right|_{p=10} = -100$$

what does $\frac{dg}{dp}$ mean?



When I raise \$
from \$10 to \$11 ...
I sell 100 fewer
gallons

3.5

Chain Rule Composite Functions

$$f(x) = x^2$$

$$g(x) = x^3$$

$$(f \circ g)(x) = f(g(x)) = (x^3)^2$$

$$= x^6$$

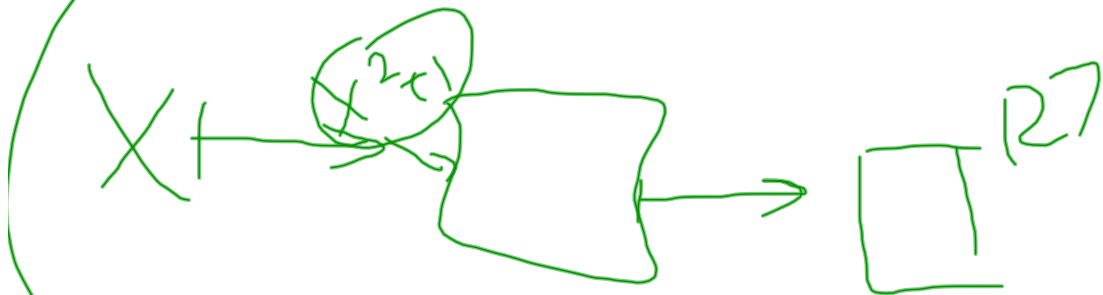
Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$f'(x) = 2(x^3) \cdot 3x^2 = 6x^5$$

$$\frac{d}{dx}(x^6) = 6x^5$$

$$\frac{d}{dx} \left((x^2+1)^{127} \right)$$



$$f(x) = x^{127}$$

$$g(x) = x^2 + 1$$

$$\frac{d}{dx} () = 127 (x^2+1)^{126} \cdot 2x$$

